Lessons 1.3.1, 1.3.2, 1.3.3 -- **Identify angle pair relationships including those created by parallel lines crossed by a transversal and solve problems using those relationships.**
Lesson 1.3.4 -- Prove the Triangle Angle Sum Theorem* and use it to solve problems.

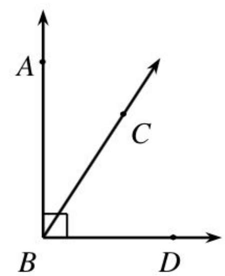
1.3.1 What is the relationship?

Angle Pair Relationships

1-68

<u>Complementary <’s</u> = sum of **90˚**

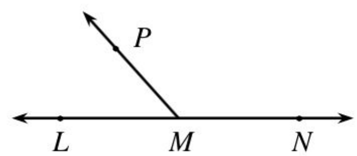
- Two angles whose measures have a sum of 90˚
- <ABC + <CBD = 90˚



Ex) <ABC and <CBD are complementary.
If <CBD = 76˚, what is the measure of <ABC?

<u>Supplementary <’s</u> = sum of **180˚**

- Two angles whose measures have a sum of 180˚
- <LMP + <PMN = 180˚
- <LMP and <PMN are also called a ***<u>straight angle pair</u>*** or ***<u>linear pair</u>***

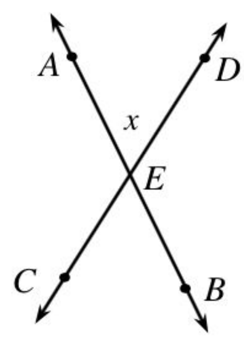


Ex) <LMP and <PMN are a linear pair.
If <LMP = 62˚, what is the measure of <PMN?

1-69

<u>Proof of vertical angle Relationships:</u>

- *<u>Conjecture:</u>* an educated guess
- *<u>Theorem:</u>* a proven conjecture



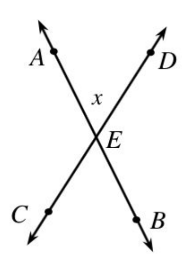
<u>Vertical <’s</u> lie on opposite sides of the intersection point.

- <AED and <CEB are vertical angles
- <AEC and <DEB are vertical angles.

<u>Congruent <’s</u> = <’s with equal measures.

- When angles have equal measure, they are considered ***<u>congruent</u>***.

Based on the figure below:

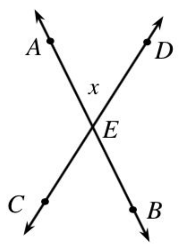


1. If x=23, determine the measures of <CEB, <AEC and <DEB. Show your work.

m<CEB = ________

m<AEC = ________

m<DEB = ________

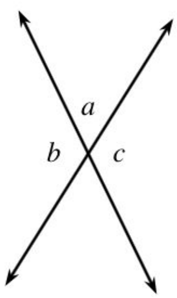


2. Which pairs of angles are <u>congruent</u>?

3. Which pairs of angles are <u>vertical pairs</u>?

Make a conjecture: Based on your answers from #2-3, can you come up with a conjecture about <u>vertical angles?</u>

1-70 & 1-71



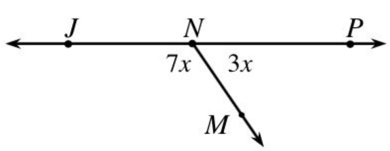
<u>Proving vertical <'s are congruent:</u>

Using your knowledge about supplemental <'s, prove that vertical <'s are congruent.

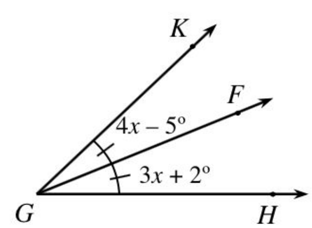
1-71

For each diagram below, identify an angle pair relationship, and use the angle pair relationship to write and solve an equation. The diagrams are not drawn to scale.

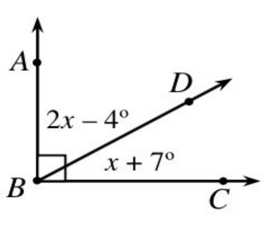
A. What is $m\angle MNP$?



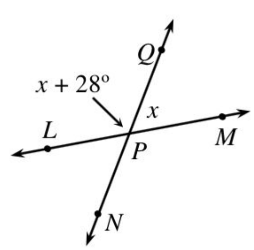
B. What is $m\angle FGH$?



C. What is $m\angle DBC$?

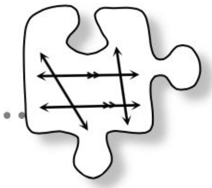


D. What is $m\angle NPM$?



1.3.2 What is the relationship?

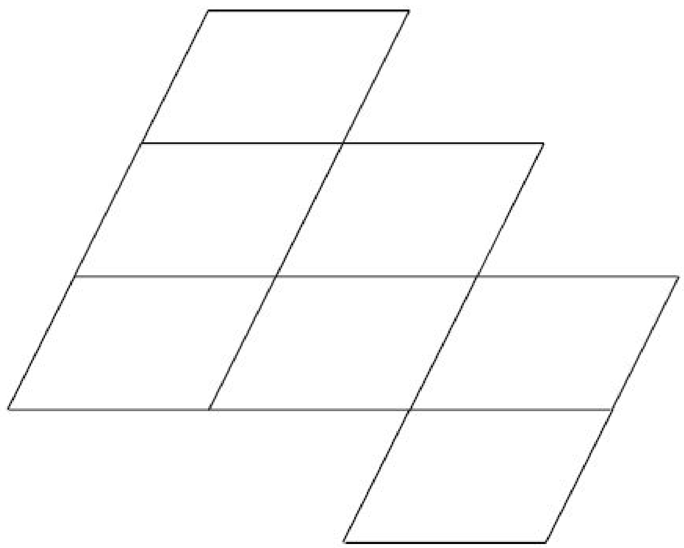
Angles Formed by Transversals



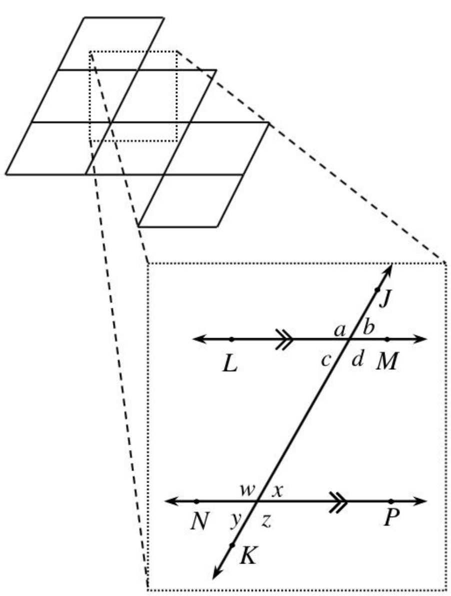
1-81

a) Based on what you learned about ***vertical angles,*** determine which angles must be congruent and label them in color. (Tip: you will need two colors.)

b) Determine which lines must be parallel and label them with tick marks.



1-82 & 1-83



A ***transversal*** a line that crosses two or more lines.

Which line is the ***transversal***? __________

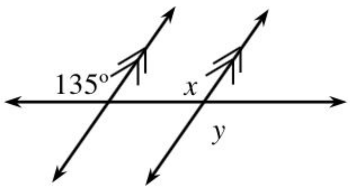
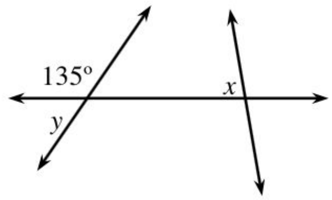
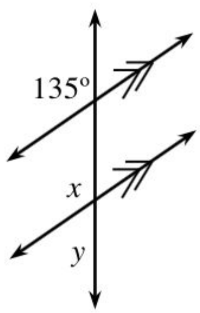
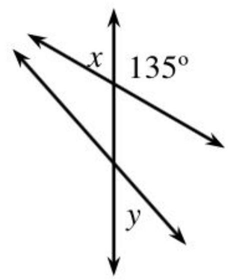
Which two lines are parallel? __________ & ___________

Corresponding <'s are congruent because they are in the same position at two different intersections of the transversal. Name the pairs of corresponding <'s.

______ & ______, ______ & ______ , ______ & ______ , ______ & ______

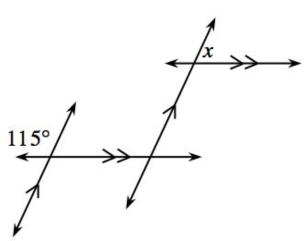
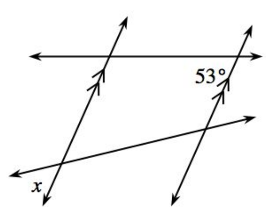
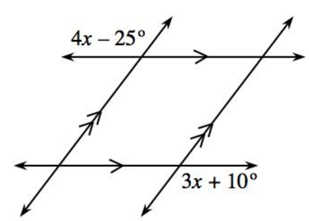
Suppose < *b* = 60˚. What do you know about angle pair relationships that could help you determine the measures of all other angles? What are the measures of the angles?

1-84 to 1-85 - *Make a conjecture:* Think about the following diagrams. Are corresponding <'s always congruent? If not, when are they congruent? In each diagram below, mark and color-code the angles that are congruent, and leave blank the angles about which you are unsure and explain why you are unsure about them. Then, determine the value of x *if possible*.

a) 	b) 
c) 	d) 

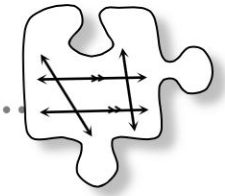
Write two *conjectures*, one about ***vertical angles*** and one about ***corresponding angles***, based on your observations from the diagrams above. Remember: A *conjecture* is written *"If… then…"*

1-86 For each diagram below, determine the value of x, if possible.

a)  x = ______________	b)  x = ______________	c)  x = ______________

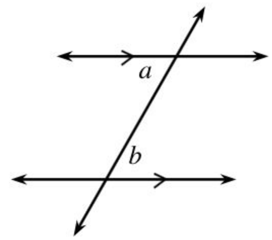
1.3.3 What is the relationship?

More Angles Formed by Transversals



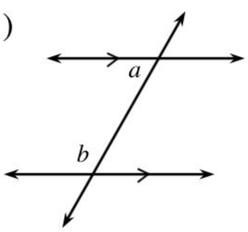
1-93

Suppose <*a* in the diagram measures 48˚. What is the measure of <*b*? Explain your thinking.

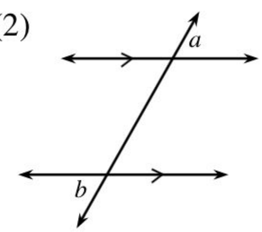


Use tracing paper to determine if the following angle pairs are congruent or supplementary. Is the pair of angles created after the translation a vertical pair or do they form a straight line?

(1)



(2)



(3)

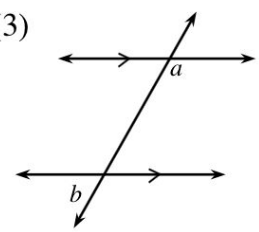
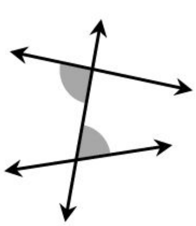


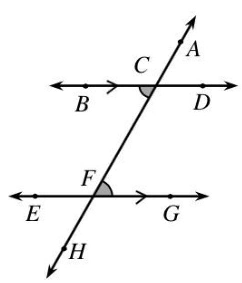
Figure 1:   Figure 2:   Figure 3:

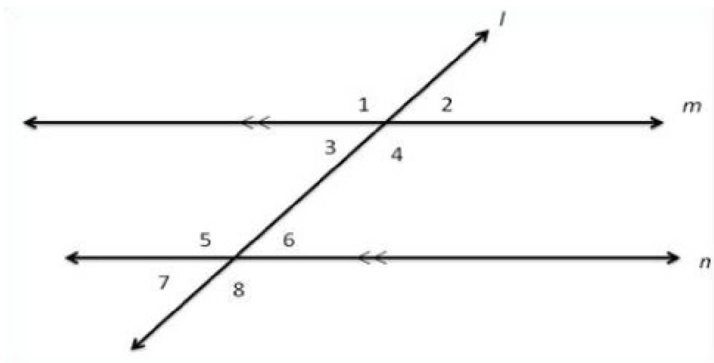
1-94

ALTERNATE INTERIOR ANGLES THEOREM

Angles between a pair of lines that are on opposite sides of a transversal are ***alternate interior angles.*** If the lines intersected by the transversal are parallel, then the alternate interior angles are congruent. Conversely, if the alternate interior angles are congruent, then the two lines intersected by the transversal are parallel.







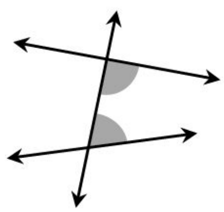
Name all congruent ***alternate interior <*** pairs in the diagram and color them.

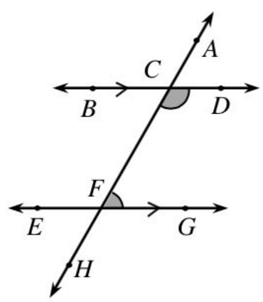
_______ & ______ , ______ & ______

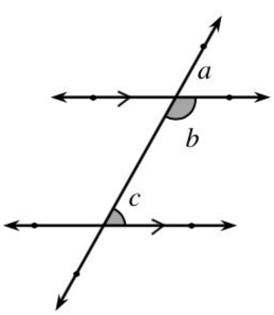
1-95

SAME SIDE INTERIOR ANGLES THEOREM

Two angles between two lines and on the same side of a transversal are ***same-side interior angles***. If the two lines cut by the transversal are parallel, then the two angles are supplementary (add up to 180°). Conversely, if the two angles are supplementary, then the two lines that are cut by the transversal are parallel.



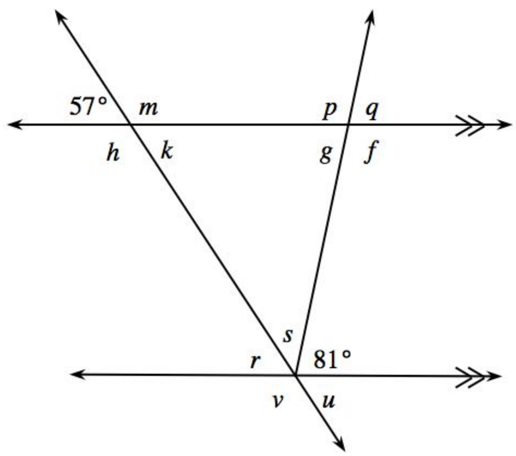




$a + b = 180º$ and $a = c$. How do you know this? (Tip: Think about ***supplementary angles***, ***corresponding angles***, and ***vertical angles*** from lesson 1.3.1 & 1.3.2) Explain why same-side interior angles are always supplementary whenever lines are parallel.

1-97

Work with your team to determine the measures of all the labeled angles using the angle relationship theorems you have learned as justification.



< *m* = ________ / Justification: ___________________________________

< *h* = _________ / Justification: ___________________________________

< *k* = _________ / Justification: ___________________________________

< *p* = _________ / Justification: ___________________________________

< *q* = _________ / Justification: ___________________________________

< *g* = _________ / Justification: ___________________________________

< *f* = __________ / Justification: ___________________________________

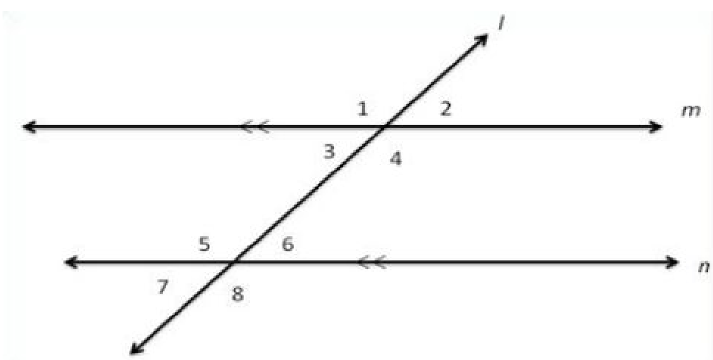
< *r* = ___________ / Justification: ________________________________

< *s* = ___________ / Justification: ________________________________

< *v* = ___________ / Justification: ________________________________

< *u* = ___________ / Justification: ________________________________

NOTES:



line *l* = ____________________

line *m* ll line *n*. (ll means parallel.)

Vertical Angles: _________________________________

____

_____ & _____, _____ & _____,

_____ & _____, _____ & _____

Corresponding Angles: _________________________

____

_____ & _____, _____ & _____,

_____ & _____, _____ & _____

Same-Side Interior Angles: ___________________________

____

_____ & _____, _____ & _____,

Alternate Interior Angles: _______________________

____

_____ & _____, _____ & _____

Same-Side Exterior Angles: ___________________________

____

_____ & _____, _____ & _____,

Alternate Exterior Angles: _______________________

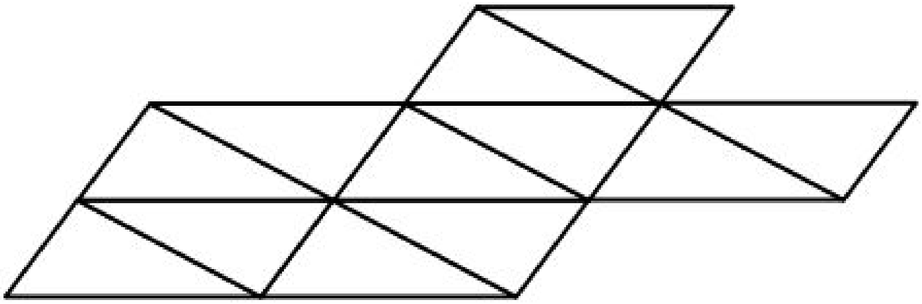
____

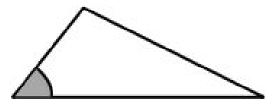
_____ & _____, _____ & _____

1.3.4 What other relationships can I find?

Angles and Sides of a Triangle

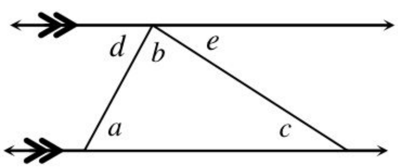
1-104





a). Determine which angles in the above tiling are congruent. You can use your own thinking or trace paper if it helps. Color each sets of congruent angles. When you are finished, every angle in your tiling should be shaded with one of the three colors.

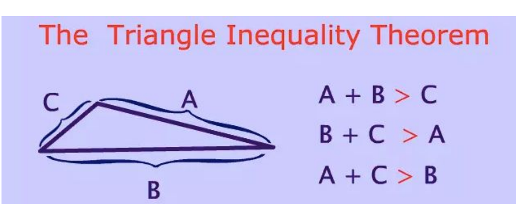
c). Examine your colored tiling. Highlight the perimeter of one triangle in the group. What do you notice about the colored angle measures inside of the triangle that also matches the same-side angle triplets?



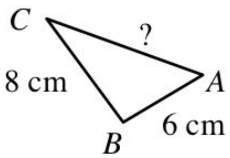
"If a polygon is a triangle, then the measures of its interior angles…

TRIANGLE ANGLE-SUM THEOREM:
The sum of the measures of the interior angles in any triangle is 180°.

1-105
TRIANGLE INEQUALITY THEOREM:



The sum of any two sides of a triangle must be greater than the third side, and the difference of two sides must be less than the third side.



Consider: If a triangle has sides of length 6 cm and 8 cm, what do you know about the length of the third side? What are the largest and smallest possible lengths for the third side? Why?

Which of the sets of side lengths below can be used to build a triangle? Explain your answers.

(1) 5 cm, 3 cm, and 10 cm

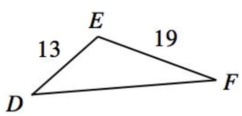
(2) 4 cm, 12 cm, and 9 cm

(3) 5 cm, 2 cm, and 4 cm

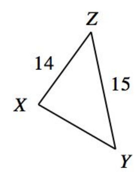
(4) 3 cm, 5 cm, and 8 cm

1-108

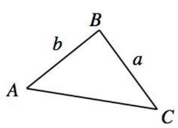
Determine the minimum and maximum lengths for each missing side in the triangles below.



_______ < DF < _______



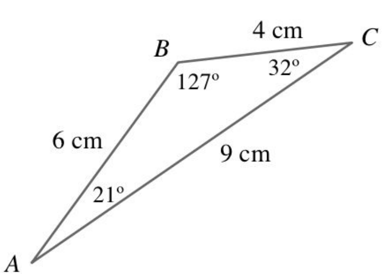
______ < XY < _______



_______ < AC < _______

1-106

LONGEST SIDE, LARGEST ANGLE CONJECTURE:



a). Side *AC* is opposite $\angle B$. What do you notice about the length of *AC* compared to the other sides of ΔABC? What is the relationship of $m\angle B$ with the other angle measures in ΔABC?

b). ΔABC has side lengths $AB = 4$ cm, $BC = 9$ cm, and $AC = 12$ cm, then which angle has the largest measure? The smallest?

c). ΔABC has side lengths $AB = 10$ cm, $BC = 10$ cm, and $AC = 12$ cm, then which angle has the largest measure? The smallest? Use your tool to investigate and answer the questions. What kind of triangle is ΔABC?

1-107

Triangle *EDF* is a right triangle with $m\angle D = 90°$. What is the longest side of the triangle? Why?