### 2.2.1 What do these shapes have in common?

## Dilations



A dilation enlarges or reduces a figure while maintaining its shape.

## 2-47

Refer to the diagrams on the reverse of this page. Each team member will choose a different number of knots and/or rubber bands. A rubber band chain is stretched from the origin so that the first knot traces the perimeter of the original polygon. Dilate the polygon from the origin by imagining a chain of $2,3,4$, or 5 rubber bands (or one rubber band with $x$ knots) to form $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$.


Diagram \#1


1. On diagram \#1 (on the reverse of this page) each team member will use the rubber-band method to dilate your polygon from the origin.
2. Trace your dilated polygon onto tracing paper.
3. Compare the dilations. Compare all side lengths and angle measures. Are the shapes congruent?
4. On diagram \#2 (on the reverse of this page) dilate the image by a scale factor of 3 .
5. Do your observations from diagram \#1 still apply? What conjectures can you make about dilating a polygon?

Diagram \#1


## Diagram \#2



## 2-48

Similar Polygons: polygons that look alike, but may be different sizes. They can be mapped onto each other with a sequence of rigid transformations and dilation. Their corresponding angles are congruent and the lengths of the corresponding sides are proportional.

Determine if each set of polygons are proportional. Use trace paper to confirm your conjecture.


2-49
The triangles below are drawn to scale. Determine if they are

similar and explain

your thinking.

Which of the following statements are correctly written and which are not? (Tip: more than one statement may be correct.)
i. $\triangle D O G \sim \triangle C A T$
ii. $\triangle D O G \sim \triangle C T A$
iii. $\triangle O G D \sim \triangle A T C$
iv. $\triangle D G O \sim \Delta C A T$

If the larger polygon is a dilation of the smaller polygon, what relationships do the corresponding angles and sides have?

What characteristics of the larger polygon would you verify to determine that it is indeed a dilation of the smaller polygon?

Where is the point of dilation?

### 2.2.2 How can I maintain the shape?

Similarity


## $\mathbf{2 - 5 1} \& 2-52$

Plot triangle $A B C$ with vertices $A(0,0), B(3,4)$, and $C(3,0)$ on graph paper. Using the origin as the point of dilation, enlarge it by a factor of 2 (imagine using two rubber bands). Label this new triangle $A^{\prime} B^{\prime} C^{\prime}$.


What are the side lengths of the enlarged triangle, $\Delta A^{\prime} B^{\prime} C^{\prime}$ ?
What are the side lengths of the original triangle, $\triangle A B C$ ? (Tip: Pythagorean Theorem or distance formula)

Calculate the area and the perimeter of $\Delta A^{\prime} B^{\prime} C^{\prime}$.
Perimeter:

Which side of $\Delta A^{\prime} \quad B^{\prime} \quad C^{\prime} \quad$ corresponds to $C B$ ?

Which side corresponds to $A B$ ?

Why does $A^{\prime} B^{\prime}$ lie directly on $A B$ and $A^{\prime} C^{\prime} \quad$ lie directly on $A C$, but $B^{\prime} C^{\prime}$ does not lie directly on $B C$ ?

Could you determine the side lengths of $\triangle A^{\prime} B^{\prime} C^{\prime}$ by adding the same amount to each side of $\triangle A B C$ ? Try this, and explain what happens.
$\triangle A B C$ is dilated until $A$ " $B^{\prime \prime}$ is 20 units long. How many times longer than $A B$ is $A^{\prime \prime} B^{\prime \prime}$ ? How long is $B^{\prime \prime} C^{\prime \prime}$ ? Show how you know.

Zoom on a copy machine works like scale factor, except zoom is often written as a percent, while scale factor is usually written as a decimal or fraction. A $200 \%$ zoom is the same as a scale factor of 2 . If the zoom is set to $50 \%$, or the scale factor to 0.5 , the machine would shrink the side lengths in half. (A scale factor of 1 would mean the figures are identical.)


Since the scale factor multiplies each side of the original polygon, then the ratio of the widths must equal the ratio of the heights. Verify that the ratios to the left are correct and equal.

A proportion is an equation with two equivalent ratios. Enlarge this figure to be 72 " wide, as in the figure at left. If $x$ is the height, write and solve an equation to determine how tall the figure must be.

In order to shrink the original figure, so that the height is 2 cm , what scale factor should be used?

## 2-59



You could also redraw the triangles separately like this:


(These are the same triangles as on the previous page.)

Write a proportion and solve for x using the corresponding sides.

Write a different proportion and solve. Did you get the same value for $x$ ?


How long is $A C$ ? How long is $A C^{\prime}$ ?

What is the ratio of the original segment $B C$ to its image $B^{\prime} C^{\prime} \quad$ ? Explain.

What is the relationship between $\overleftrightarrow{B^{\prime} C^{\prime}}$ and $\overleftrightarrow{B C}$ in the original diagram?

## 2-60

There is a $60^{\circ}$ angle shown in Diagram \#1 at right. A student extended the sides of the angle so they were twice as long, as shown in Diagram \#2. "Therefore, the new angle must measure $120^{\circ}$," he explained. Do you agree? Discuss this with your team and write a response to Al.


Diagram \#1


Diagram \#2

