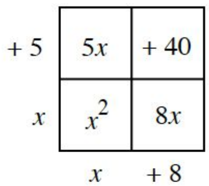
4.1.1 How can I determine the product?

Introduction to Factoring Expressions

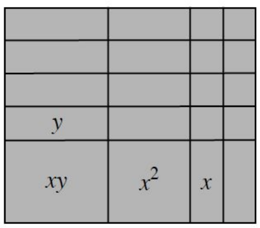
Example:



Area as a product *(factored form)*: $(x + 5)(x + 8)$

Area as a sum: $x^2 + 13x + 40$

4-1 - Review area models



a). Write an equation showing that the area of the product is equal to the area of the sum.

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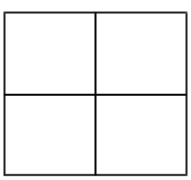
b). Use algebra tiles and an area model to multiply $(6x - 1)(3x + 2)$

Area Model:



Algebra Tile Area Model:

4-2 - *Factoring:* To factor an expression is to write it as a product. When an expression is written as a *product*, it is in ***factored form***. Each of the expressions being multiplied is called a ***factor***.

Explore: Can every expression be factored? (Can you rewrite every sum as a product?)

For each expression below, draw an area model and an algebra tile model for each expression, then write an equation showing that the area as a sum equals the area as a product. (If you cannot build a rectangle with the algebra tiles, the expression cannot be factored.)

a. $2x^2 + 7x + 6$

Area Model

Algebra Tile Area Model:

Equation: ______________________________

b. $6x^2 + 7x + 2$

Area Model

Algebra Tile Area Model:

Equation: ______________________________

c. $x^2 + 4x + 1$

Area Model

Algebra Tile Area Model:

Equation: ______________________________

d. $2xy + 6x + y^2 + 3y$

Area Model

Algebra Tile Area Model:

Equation: ______________________________

Explore: Can you find any special strategies that can help you determine the dimensions of the rectangle? Write the sum and product for the following area models.

a) Area Model

Algebra Tile Area Model:

$2x$	5
$6x^2$	$15x$

a) Equation: ____

b) Area Model

Algebra Tile Area Model:

$-2y$	-6
$5xy$	$15x$

b) Equation: ____

c) Area Model

Algebra Tile Area Model:

$-9x$	-12
$12x^2$	$16x$

c) Equation: ____

4-4 - Casey

$2x$	5
$6x^2$	$15x$

a) Describe a pattern Casey may have found. (Draw, highlight, show it!)

b) *Explore:* Does Casey's pattern always work? Check whether her pattern works for all of the 2 x 2 area models in problem 4-3.

4.1.2 Is there a faster method?

Factoring with Area Models

4-12 - Casey's Pattern (continue from problem 4-4)

$-35x$	14
$10x^2$	$-4x$

Equation: ____

Does the area model above fit Casey's pattern for diagonals? Explain your answer.

4-13 - Factoring Quadratic Expressions

A polynomial in the form $\boldsymbol{ax^2 + bx + c}$, **with** $\boldsymbol{a \neq 0}$, is called a ***quadratic expression*** in ***standard form***.

Factor: $2x^2 + 5x + 3$

Area Model

Algebra Tile Area Model:

Equation: ____

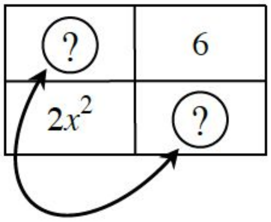
Miguel's model: Finish the rectangle by deciding how to split and place the remaining term.

Factor: $3x^2 + 10x + 8$

	8
$3x^2$	

Area as a product (factored form): ________________________

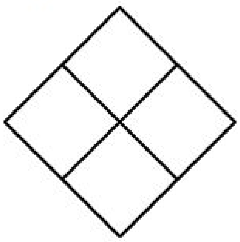
Kelly's model: $2x^2 + 7x + 6$



Without actually factoring yet, what do you know about the missing two parts of the area model?

To complete Kelly's area model, create and solve a Diamond Problem.

product



sum

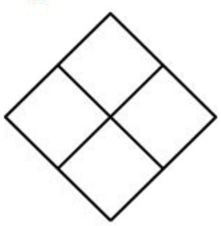
Use your results from the Diamond Problem to complete the area model for $2x^2 + 7x + 6$, and then write the area as a product of factors.

Area as a product of factors:

4-14 & 4-15 - Using an area model is convenient when the numbers become too larage to manage with algebra tiles. Use a diamond problem to factor:

$6x^2 + 17x + 12$

product

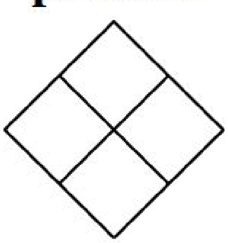


sum

Area as a product of factors:

$7x^2 + 10x + 3$

product



sum

Area as a product of factors:

4.1.3 How can I factor this?

Factoring More Quadratics

4-24 - If possible, factor each quadratic expression below. Use a diamond problem and an area model for each one.

a) $x^2 + 6x + 9$

Area as a product of factors:

____

b) $2x^2 + 5x + 3$

Area as a product of factors:

____

c) $x^2 + 5x - 7$

Area as a product of factors:

____

d) $3m^2 + m - 14$

Area as a product of factors:

____

4-25 - You have been working with quadratic expressions written in the form $ax^2 + bx + c$. But what if a term is missing or in a different order?

a) $9x^2 - 4$

Area as a product of factors:

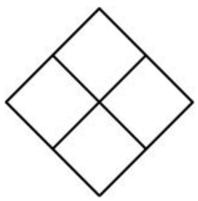
____

b) $12x^2 - 16x$

Area as a product of factors:

____

c) $3 + 8k^2 - 10k$



Area as a product of factors:

____

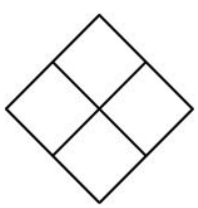
d) $40 - 100m$

Area as a product of factors:

____

4-26 - Now use an area model and a Diamond Problem to factor the expression below. Compare your answer with your teammates' answers. Is there more than one possible answer?

$4x^2 - 10x - 6$



Area as a product of factors:

____

4-27 - Emily Rae designed an area model puzzle for her team to solve. Write an equation that shows the area of the entire rectangle as a product equal to its area as sum.

	$x - 2$	
$x + 7$		
	$3x^2 - 5x - 2$	$6x^2 + 5x + 1$

Equation: ____

4.1.4 Can it be factored further?

Factoring Completely

A polynomial is ***factored completely*** if none of the resulting factors can be factored further using integer coefficients. For example, $-2(x + 3)(x - 1)$ is the completely factored form of $-2x^2 - 4x + 6$. Also, 12 factors completely into 2 x 2 x 3, because the factors are all prime numbers.

4-34 - Factor the following expressions. (Remember the area model and diamond problems from previous lessons.)

a) $9x^2 - 12x + 4$

b) $81m^2 - 1$

c) $28 + x^2 - 11x$

d) $3n^2 + 9n + 6$

4-35 - Comparisons

a) Is there more than one way to factor $3n^2 + 9n + 6$? Why or why not?

b) Why does $3n^2 + 9n + 6$ have more than one factored form while the other quadratics in problem 4-34 only have one possible factored form? Look for clues in the original expression and in the different factored forms.

c) *Without factoring*, predict which quadratic expressions below may have more than one factored form.

i) $12t^2 - 10t + 2$

ii) $5p^2 - 23p + 10$

iii) $10x^2 + 25x - 15$

iv) $3k^2 + 7k - 6$

4-36 - Factoring Completely

In the equation $12t^2 - 10t + 2$, notice that each term is divisible by 2. That is, each term has a ***common factor*** of 2. (A common factor is a factor that is the same for two or more terms. For example, x^2 is a common factor of $3x^2$ and $-5x^2y^2$).

An expression is ***factored completely*** if none of the factors can be factored further.

a-b) Rewrite the expression $10x^2 + 25x - 15$ with the common factor factored out. Then, factor the expression completely.

4-37 - Factoring Completely - Factor the following expressions completely.

a) $5x^2 + 15x - 20$

b) $3x^3 - 6x^2 - 45x$

c) $2x^2 - 50$

d) $x^2y - 3xy - 10y$

4.1.5 How can I use the structure?

Factoring Special Cases

4-45 - Special Quadratics

You have been given cards with the following quadratic equations on them. Factor them completely if possible, and then write answers on this paper as well as on the cards. Then, with your group sort them into groups based on patterns.

a) $x^2 - 49$
b) $x^2 + 2x - 24$
c) $x^2 - 10x + 25$
d) $9x^2 + 12x + 4$
e) $5x^2 - 4x - 1$
f) $4x^2 - 25$
g) $x^2 - 6x + 9$
h) $x^2 - 36$
i) $7x^2 - 20x - 3$
j) $4x^2 + 20x + 25$
k) $x^2 + 4$
l) $9x^2 - 1$

What are some of the patterns you recognized? What makes these quadratics “special?”

4-46
Which of the following quadratic equations fit the patterns you recognized in problem 4-45? Show all work, and explain your answers.

a) $25x^2 - 1$

b) $x^2 - 5x - 36$

c) $x^2 + 8x + 16$

d) $9x^2 - 12x + 4$

e) $9x^2 + 4$

f) $9x^2 - 100$

4-47 - Why do these patterns work?
a) Difference of Squares: A polynomial that can be factored as the product of the sum and difference of two terms. The general pattern is $x^2 - y^2 = (x + y)(x - y)$. For example, the difference of squares $4x^2 - 9$ can be factored as $(2x + 3)(2x - 3)$. Complete the area model below for the quadratic expression $u^2 - w^2$:

b) Perfect Square Trinomial: Trinomials of the form $a^2x^2 + 2abx + b^2$, where a and b are nonzero real numbers, are known as perfect square trinomials and factor as $(ax + b)^2$. For example, the perfect square trinomial $9x^2 - 24x + 16$ can be factored as $(3x - 4)^2$. Complete the area model below for the quadratic expression $u^2 + 2uw + w^2$
